An Alternate Solution

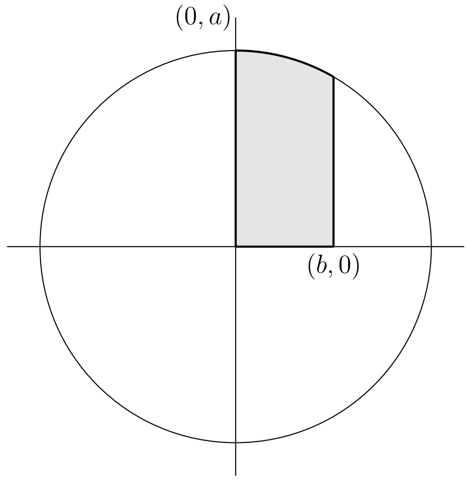

Figure 1: The area of the shaded region is $\int_0^b \sqrt{a^2 - x^2}\,dx$.

As Professor Miller explained in lecture, the area of the region shown in Figure 1 is $\int_0^b \sqrt{a^2 - x^2}\,dx$. Use the substitution $x = a\cos\theta$ to solve this integral. Hint: pay particular attention to your limits of integration.

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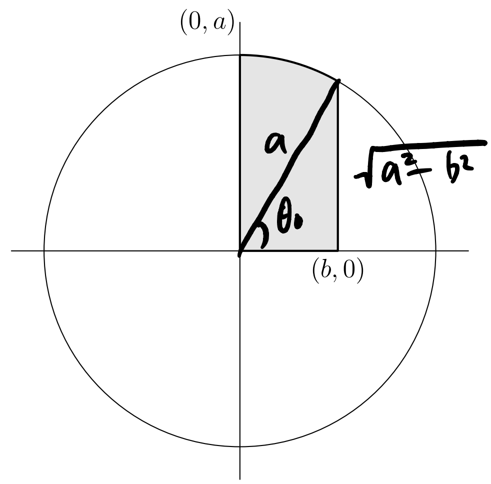

Figure 1: The area of the shaded region is $\int_0^b \sqrt{a^2-x^2}\,dx$.

As Professor Miller explained in lecture, the area of the region shown in Figure 1 is $\int_0^b \sqrt{a^2-x^2}\,dx$. Use the substitution $x = a\cos\theta$ to solve this integral. Hint: pay particular attention to your limits of integration.

$$\int \sqrt{a^2-x^2}\,dx$$

$$x = a\cos\theta, \quad dx = -a\sin\theta\,d\theta$$

$$= \int \sqrt{a^2-a^2\cos^2\theta}\,(-a\sin\theta\,d\theta)$$

$$= \int a\sin\theta(-a\sin\theta)\,d\theta$$

$$= -a^2\int \sin^2\theta\,d\theta$$

$$= -a^2\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)+C$$

$$= -a^2\left(\frac{\theta}{2}-\frac{\sin\theta\cos\theta}{2}\right)+C$$

$$= -\frac{a^2}{2}\left(\arccos\frac{x}{a}-\frac{x\sqrt{a^2-x^2}}{a^2}\right)+C$$

$$\int \sin^2\theta\,d\theta = \int \frac{1-\cos 2\theta}{2}\,d\theta$$

$$= \frac{\theta}{2}-\frac{\sin 2\theta}{4}+C$$

$$\int_0^b \sqrt{a^2-x^2}\,dx = -\frac{a^2}{2}\arccos\frac{x}{a}+\frac{x\sqrt{a^2-x^2}}{2}\Bigg|_0^b$$

$$= -\frac{a^2}{2}\arccos\frac{b}{a}+\frac{b\sqrt{a^2-b^2}}{2}-\left(-\frac{a^2}{2}\frac{\pi}{2}\right)$$

$$= \frac{a^2}{2}\left(\frac{\pi}{2}-\theta_0\right)+\frac{1}{2}b\sqrt{a^2-b^2}$$

$$\theta_0 = \arccos\frac{b}{a}$$